

DISCLAIMER

Ich übernehme keine Haftung über mögliche Fehler in den Notizen. Es hat sicherlich ein paar drinnen.

Fehler können per Mail an jirruh@ethz.ch gemeldet werden.

Serie 1

Aufgabe 1

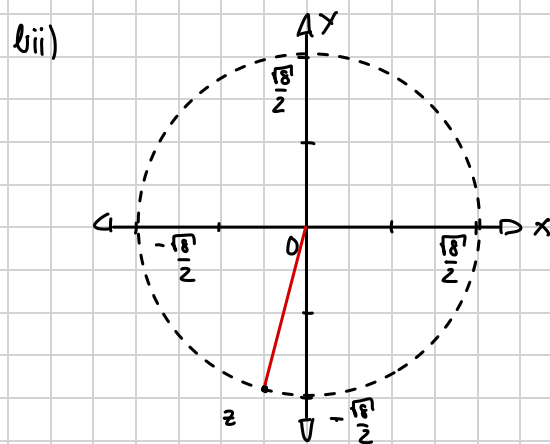
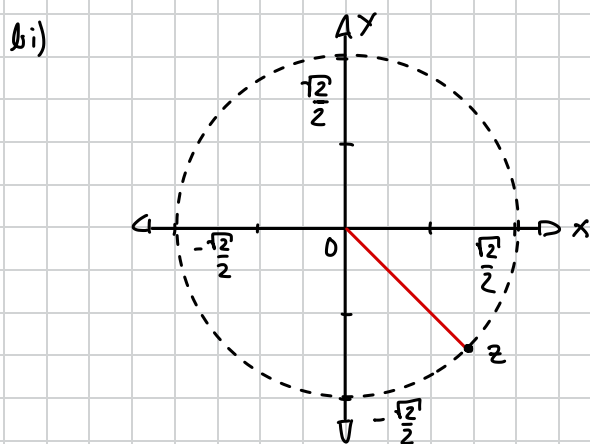
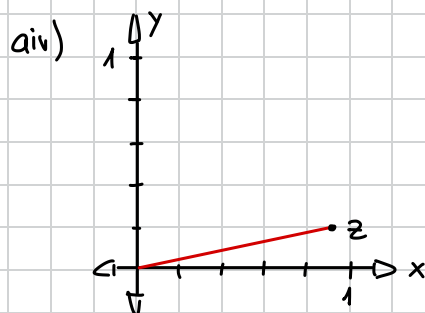
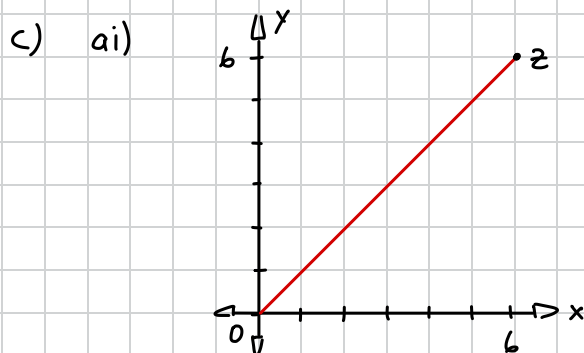
a) i) $6 + 6i$ ii) $-10 + i$ iii) $3 - 39i$

iv) $\frac{(12 + 3i)}{(12 + 6i)} = \frac{(12 + 3i)(12 - 6i)}{144 + 36} = \frac{162 - 36i}{180}$

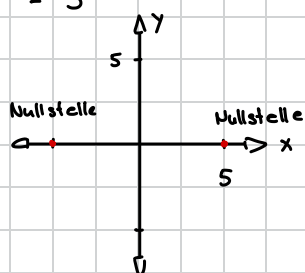
b) i) $-i \Rightarrow 1(\cos(-\frac{\pi}{2}) + i \sin(-\frac{\pi}{2}))$
 $1 - i \Rightarrow \sqrt{2}(\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4}))$ } $|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$
 $\arg(z) = (-\frac{\pi}{2} + \frac{\pi}{4}) = -\frac{\pi}{4}$
 $\Rightarrow z = \frac{\sqrt{2}}{2}(\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4}))$

ii) $-2 - 2i \Rightarrow \sqrt{8}(\cos(-\frac{3\pi}{4}) + i \sin(-\frac{3\pi}{4}))$
 $1 + \sqrt{3}i \Rightarrow 2(\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))$ } $|z| = \sqrt{8}/2$
 $\arg(z) = (-\frac{3\pi}{4} - \frac{\pi}{3}) = -\frac{13\pi}{12} = \frac{11\pi}{12}$
 $\Rightarrow z = \frac{\sqrt{8}}{2}(\cos(\frac{11\pi}{12}) + i \sin(\frac{11\pi}{12}))$

iii) $1 + \sqrt{3}i \Rightarrow 2(\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))$
 $\sqrt{3} + i \Rightarrow 2(\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6}))$ } $|z| = 1$
 $\arg(z) = (\frac{\pi}{3} - \frac{\pi}{6}) = \frac{\pi}{6}$
 $z = 1(\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6}))$



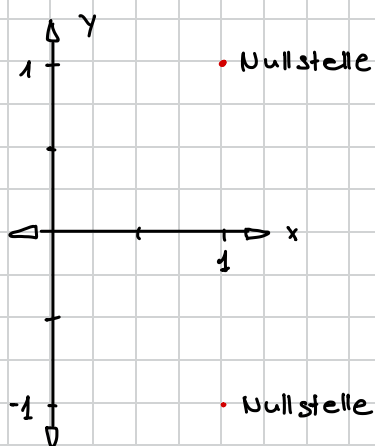
A diagram of the complex plane with a dashed unit circle centered at the origin. The horizontal axis is labeled x and the vertical axis is labeled y . The origin is marked with 0 . The unit circle has tick marks at 1 and -1 on the x -axis, and 1 and -1 on the y -axis. A point z is marked on the unit circle in the first quadrant, and a red line segment connects the origin to z .

$$i) \quad z = \pm 5$$


$$\text{ii) } z^2 - 2z + 2 \stackrel{!}{=} 0$$

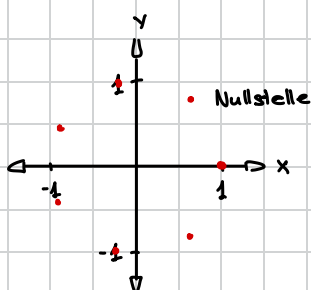
$$z_{1,2} = 1 \pm \sqrt{1 - 2}$$

$$= 1 \pm i$$



iv) $z = 1(\cos(71) + i \cdot \sin(71))$

$$Tf = 2\pi i n \Rightarrow f = \frac{2\pi i n}{T}$$



iii) $z^3 + z^2 - 2 : z - 1 = z^2 + 2z + 2$

$$z^3 - z^2$$

✓	$2z^3$
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$$2z^2 - 2z$$

$$/ 27^{-2}$$

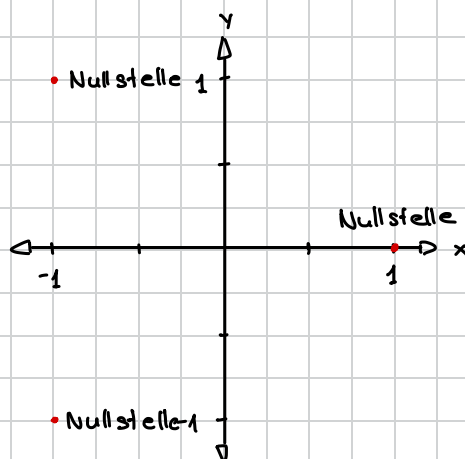
$2x - 2$

$$z_1 = 1$$

$$z^2 + 2z + 2 = 0$$

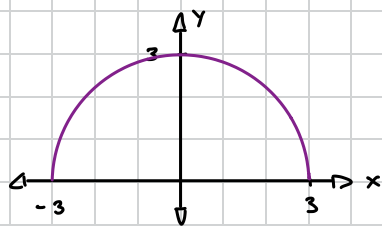
$$z_{2,3} = -1 \pm \sqrt{1 - 2}$$

$$= -1 \pm i$$



Aufgabe 1

i) $M = \{z \in \mathbb{C} \mid |z| = 3, \operatorname{Im}(z) \geq 0\}$



ii) $M = \left\{ z \in \mathbb{C} \mid \frac{|z+2-2i|}{|z+i|} = 2 \right\}$
 $= \left\{ z \in \mathbb{C} \mid \frac{|x+2+i(y-2)|}{|x+i(y+1)|} = 2 \right\}$

$$(|x+2+i(y-2)|)^2 = (2|x+i(y+1)|)^2$$

$$(x+2)^2 + (y-2)^2 = 4(x^2 + (y+1)^2)$$

$$x^2 + 4x + 4 + y^2 - 4y + 4 = 4x^2 + 4y^2 + 8y + 4$$

$$-3x^2 + 4x + 4 = 3y^2 + 12y$$

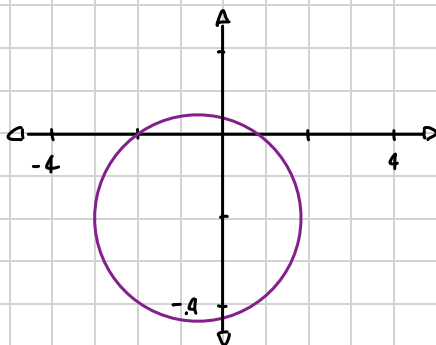
$$3x^2 - 4x + 3y^2 + 12y = 4$$

$$x^2 - \frac{4}{3}x + y^2 + 4y = \frac{4}{3}$$

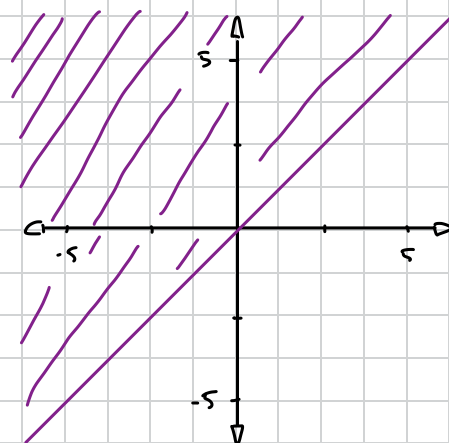
$$\left(x - \frac{2}{3}\right)^2 + (y+2)^2 = \frac{52}{3}$$

$$M = \left\{ z \in \mathbb{C} \mid \left| z - \frac{2}{3} + 2i \right| = \frac{2\sqrt{13}}{3} \right\}$$

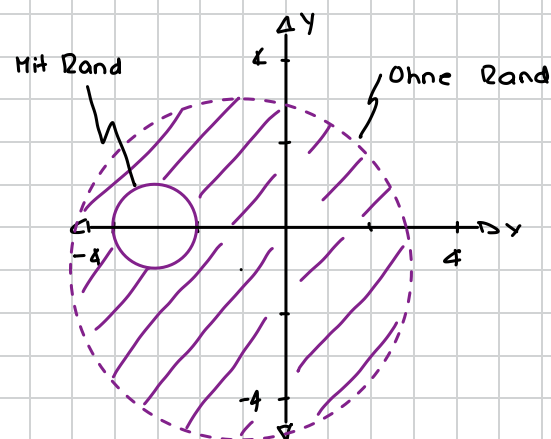
⚡ Nur der Rand



iii) $M = \{z \in \mathbb{C} \mid \operatorname{Im}(z) \geq \operatorname{Re}(z)\}$



iv) $M = \{z \in \mathbb{C} \mid |z-3| \geq 1 \text{ und } |z-1-i| < 4\}$



Serie 3

Aufgabe 1

a) i) $\lim_{n \rightarrow \infty} \cos(in) = \lim_{n \rightarrow \infty} \cosh(n) = \underline{\underline{\infty}}$

ii) $\lim_{n \rightarrow \infty} 1 + (-1)^n \cdot \frac{1}{n} = 1 + \lim_{n \rightarrow \infty} (-1^n \cdot \frac{1}{n}) = 1 + 0i = \underline{\underline{1}}$ || Wenn $n \rightarrow \infty$ wird $\frac{1}{n} \rightarrow 0$

iii) $\lim_{n \rightarrow \infty} \frac{(n + 2\pi i)^n}{n^n} = \lim_{n \rightarrow \infty} \frac{n^n}{n^n} = \underline{\underline{1}}$ || $2\pi i$ hat kein Einfluss aufs Ergebnis wenn $n \rightarrow \infty$

iv) $\lim_{n \rightarrow \infty} \text{Arg}(1 + (-1)^n \frac{1}{n})$ || $\lim_{n \rightarrow \infty} 1 + (-1)^n \frac{1}{n} = 1$
 $\text{Arg}(1) = \underline{\underline{0}}$

b) $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} + \frac{(\pi i)^n}{n!} \right) = \sum_{n=0}^{\infty} \frac{1}{2^n} + \sum_{n=0}^{\infty} \frac{(\pi i)^n}{n!} = 2 + \exp(\pi i) = \underline{\underline{2 + e^{\pi i}}}$

Aufgabe 2

$$\exp(it) = \cos(t) + i \cdot \sin(t)$$

$$\exp(z_1 + z_2) = \exp(z_1) \cdot \exp(z_2) \quad || \quad z_1 = ix ; z_2 = iy$$

$$\exp(i(x+y)) = \exp(ix) \cdot \exp(iy) \quad || \text{Eulerformel}$$

$$\exp(i(x+y)) = (\cos(x) + i \sin(x)) \cdot (\cos(y) + i \sin(y))$$

$$\cos(x+y) + i \sin(x+y) = (\cos(x) + i \sin(x)) \cdot (\cos(y) + i \sin(y))$$

$$\cos(x+y) + i \sin(x+y) = \cos(x)\cos(y) + \cos(x)i\sin(y) + i\sin(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(x+y) + i \sin(x+y) = (\cos(x)\cos(y) - \sin(x)\sin(y)) + i(\cos(x)\sin(y) + \cos(y)\sin(x))$$

$$\Rightarrow \cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\Rightarrow \sin(x+y) = \cos(x)\sin(y) + \cos(y)\sin(x)$$

|| Ich bin zu faul um es für $x-iy$ zu beweisen :-)

□

Aufgabe 3

i) $\lim_{z \rightarrow 0} \frac{\bar{z} + z^2}{z}$ || $z = x + iy$

$$\lim_{z \rightarrow 0} \frac{x - iy + x^2 + 2xyi - y^2}{x + iy} \quad || \text{Wir müssen den Grenzwert von Re und Im separat betrachten}$$

$$\lim_{y \rightarrow 0} \frac{x - iy + x^2 + 2xyi - y^2}{x + iy} \quad || x=0$$

$$\lim_{y \rightarrow 0} \frac{-iy - y^2}{iy} = \lim_{y \rightarrow 0} -1 - \frac{y^2}{iy} = \lim_{y \rightarrow 0} -1 - \frac{y}{i} = \underline{\underline{-1}}$$

Aufgabe 3

$$\lim_{x \rightarrow 0} \frac{x - iy + x^2 + 2xy - y^2}{x + iy} \quad \parallel y=0$$

$$\lim_{x \rightarrow 0} \frac{x + x^2}{x} = \lim_{x \rightarrow 0} 1 + x = 1$$

$$z_0 = 1 - i \neq 0 \quad \parallel \text{Somit existiert der Limes nicht.}$$

$$ii) \lim_{z \rightarrow 0} \frac{\cos(z) - 1}{z^2} \quad \parallel \cos(z) = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots \quad (\text{Taylorreihe})$$

$$\lim_{z \rightarrow 0} \frac{-\frac{z^2}{2!} + \frac{z^4}{4!} - \dots}{z^2} = \underline{\underline{-\frac{1}{2}}}$$

$$iii) \lim_{z \rightarrow 0} \frac{\sin(z)}{\bar{z}} \quad \parallel z = x + iy$$

$$\lim_{z \rightarrow 0} \frac{\sin(x + iy)}{x - iy} \quad \parallel \text{Wir müssen den Grenzwert von Re und Im separat betrachten}$$

$$\lim_{x \rightarrow 0} \frac{\sin(x + iy)}{x - iy} \quad \parallel y=0$$

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1 \quad \parallel \lim_{x \rightarrow 0} \frac{\sin(x)'}{x'} = \frac{\cos(x)}{1} = 1 \quad (\text{Bernoulli - l'Hôpital})$$

$$\lim_{y \rightarrow 0} \frac{\sin(x + iy)}{x - iy} \quad \parallel x=0$$

$$\lim_{y \rightarrow 0} \frac{\sin(iy)}{-iy} = \lim_{y \rightarrow 0} \frac{i \cdot \sin(y)}{-iy} = \lim_{y \rightarrow 0} -\frac{\sin(y)}{y} = -1 \quad \parallel \text{Bernoulli - l'Hôpital}$$

$$\Rightarrow z_0 = 1 - i \neq 0 \quad \parallel \text{Somit existiert der Limes nicht.}$$

Aufgabe 1

a) i) $\underline{e^i = \cos(1) + i \sin(1)}$

ii) $e^{1-2i} = e \cdot e^{-2i}$

$e^{-2i} = \cos(-2) + i \sin(-2)$

$\Rightarrow \underline{e^{1-2i} = e \cos(-2) + e \cdot i \sin(-2)}$

iii) $\underline{\text{Log}(1+i) = \log(\sqrt{2}) + i \frac{\pi}{4}}$

b) i) $\cos(10i) = \cosh(10)$

$\cosh(10) = \frac{e^x + e^{-x}}{2}$

ii) $\sin(5+5i) = \sin(5)\cos(5i) + \cos(5)\sin(5i)$

$\cos(5i) = \cosh(5) = \frac{e^5 + e^{-5}}{2}$

$\sin(5i) = \sinh(5) = \frac{e^5 - e^{-5}}{2}$

$\underline{\sin(5+5i) = \sin(5) \frac{e^5 + e^{-5}}{2} + \cos(5) \frac{e^5 - e^{-5}}{2}}$

iii) $\sin(2-i) = \sin(2)\cos(i) - \cos(2)\sin(i)$

$\cos(i) = \frac{e^1 + e^{-1}}{2}$

$\sin(i) = \frac{e^1 - e^{-1}}{2}$

$\underline{\sin(2-i) = \sin(2) \frac{e^1 + e^{-1}}{2} - \cos(2) \frac{e^1 - e^{-1}}{2}}$

Aufgabe 2

$\text{Log}(z_1 \cdot z_2) \neq \text{Log}(z_1) + \text{Log}(z_2)$

$\Rightarrow \begin{cases} z_1 = -1 \\ z_2 = i \end{cases}$

|| Wir suchen z_1 und z_2 , sodass die Summe der $\text{Arg}()$ $> \pi$ sind aber das $\text{Arg}()$ von $z_1 \cdot z_2$ kleiner als π ist.

Aufgabe 3

$$(\cos(\phi) + i \sin(\phi))^n = \cos(n\phi) + i \sin(n\phi) \quad \parallel \text{Beweis durch Induktion}$$

$$n = 0$$

$$(\cos(\phi) + i \sin(\phi))^0 = \cos(0\phi) + i \sin(0\phi)$$

$$1 = 1 + 0 \quad \checkmark$$

$$n = 1$$

$$(\cos(\phi) + i \sin(\phi))^1 = \cos(\phi) + i \sin(\phi) \quad \checkmark$$

$$n = k + 1$$

$$(\cos(\phi) + i \sin(\phi))^{k+1} = (\cos(\phi) + i \sin(\phi))^k \cdot (\cos(\phi) + i \sin(\phi))$$

$$= (\cos(k\phi) + i \sin(k\phi)) (\cos(\phi) + i \sin(\phi))$$

$$= \cos(k\phi)\cos(\phi) + \cos(k\phi)i\sin(\phi) + i\sin(k\phi)\cos(\phi) - \sin(k\phi)\sin(\phi) \quad \parallel \text{Kosinus \& Sinus Satz}$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin(A+B) = \cos(A)\sin(B) + \cos(B)\sin(A)$$

$$\Rightarrow \cos(k\phi + \phi) + i \sin(k\phi + \phi) = \cos((k+1)\phi) + i \sin((k+1)\phi)$$

□

Serie 5

Aufgabe 1

$$f'(z) = \lim_{z \rightarrow z_0} \frac{f(z) - f(z_0)}{z - z_0} = \lim_{\Delta z \rightarrow 0} \frac{f(z + \Delta z) - f(z)}{\Delta z} \quad || \Delta z = z - z_0$$

i) $f(z) = 3z^3 + z - 3$

$$f'(z) = \lim_{\Delta z \rightarrow 0} \frac{3(z + \Delta z)^3 + (z + \Delta z) - 3 - (3z^3 + z - 3)}{\Delta z}$$

$$= \lim_{\Delta z \rightarrow 0} \frac{3(z^3 + 3z^2\Delta z + 3z\Delta z^2 + \Delta z^3) + (z + \Delta z) - 3 - (3z^3 + z - 3)}{\Delta z}$$

$$= \lim_{\Delta z \rightarrow 0} \frac{\cancel{3z^3} + 9z^2\Delta z + 9z\Delta z^2 + 3\Delta z^3 + \cancel{z} + \Delta z - \cancel{3} - \cancel{3z^3} - \cancel{z} + \cancel{3}}{\Delta z}$$

$$= \lim_{\Delta z \rightarrow 0} \frac{9z^2\Delta z + 9z\Delta z^2 + 3\Delta z^3 + \Delta z}{\Delta z}$$

$$= \lim_{\Delta z \rightarrow 0} \frac{\Delta z (9z^2 + 9z\Delta z + 3\Delta z^2 + 1)}{\Delta z}$$

$$= \lim_{\Delta z \rightarrow 0} 9z^2 + 9z\Delta z + 3\Delta z^2 + 1$$

$$\Rightarrow \underline{f'(z) = 9z^2 + 1}$$

ii) $f(z) = \sin(\operatorname{Re}(z)) = \sin(x)$ || Grundsätzlich wäre diese Gleichung ableitbar aber es ist in einem Bereich ableitbar.

$$f'(z) = \cos(x)$$

$$\tilde{x} \in [\pi/2 + k\pi]$$

\Rightarrow Wenn $x \in \tilde{x}$ dann existiert die Ableitung, sonst nicht

iii) $f(z) = \frac{1}{z^2}$

$$f'(z) = \lim_{\Delta z \rightarrow 0} \frac{\frac{1}{(z + \Delta z)^2} - \frac{1}{z^2}}{\Delta z} = \lim_{\Delta z \rightarrow 0} \frac{\frac{z^2 - (z + \Delta z)^2}{z^2(z + \Delta z)^2}}{\Delta z} = \lim_{\Delta z \rightarrow 0} \frac{z^2 - (z + \Delta z)^2}{\Delta z (z^2(z + \Delta z)^2)}$$

$$= \lim_{\Delta z \rightarrow 0} \frac{\cancel{z^2} - \cancel{z^2} - 2z\Delta z - \Delta z^2}{\Delta z (z^2(z + \Delta z)^2)} = \lim_{\Delta z \rightarrow 0} \frac{\Delta z (-2z - \Delta z)}{\Delta z (z^2(z + \Delta z)^2)}$$

$$\Rightarrow \underline{f'(z) = \frac{-2z}{z^4} = -\frac{2}{z^3}}$$

$$\text{iv) } f(z) = e^{-\pi z^2}$$

$$f'(z) = \lim_{\Delta z \rightarrow 0} \frac{e^{-\pi(z+\Delta z)^2} - e^{-\pi z^2}}{\Delta z}$$

$$= \lim_{\Delta z \rightarrow 0} \frac{e^{-\pi(z^2 + 2z\Delta z + \Delta z^2)} - e^{-\pi z^2}}{\Delta z} = \lim_{\Delta z \rightarrow 0} \frac{e^{-\pi z^2} \cdot e^{-\pi(2z\Delta z + \Delta z^2)} - e^{-\pi z^2}}{\Delta z}$$

\Rightarrow Wir verwenden die Approximation, dass $e^{-\pi(2z\Delta z + \Delta z^2)} \approx 1 - \pi(2z\Delta z + \Delta z^2)$ für

$\Delta z \rightarrow 0$ (Taylor Series $\rightarrow e^x = 1 + x + \frac{x^2}{2!} + \dots$)

$$= \lim_{\Delta z \rightarrow 0} \frac{e^{-\pi z^2} (1 - \pi(2z\Delta z + \Delta z^2)) - e^{-\pi z^2}}{\Delta z}$$

$$= \lim_{\Delta z \rightarrow 0} \frac{e^{-\pi z^2} (-\pi(2z\Delta z + \Delta z^2))}{\Delta z}$$

$$\Rightarrow \underline{f'(z) = -2\pi z \cdot e^{-\pi z^2}}$$

Aufgabe 2

$$g(t) = f(\gamma(t))$$

$$g'(t) = \lim_{\Delta t \rightarrow 0} \frac{g(t+\Delta t) - g(t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{f(\gamma(t+\Delta t)) - f(\gamma(t))}{\Delta t} \quad \parallel \quad \frac{\gamma(t+\Delta t) - \gamma(t)}{\gamma(t+\Delta t) - \gamma(t)}$$

$$= \lim_{\gamma(t+\Delta t) \rightarrow \gamma(t)} \frac{f(\gamma(t+\Delta t)) - f(\gamma(t))}{\gamma(t+\Delta t) - \gamma(t)} \cdot \lim_{\Delta t \rightarrow 0} \frac{\gamma(t+\Delta t) - \gamma(t)}{\Delta t} \quad \parallel \quad \Delta \gamma = \gamma(t+\Delta t) - \gamma(t) = 0 \Delta z = z - z_0$$

$$\Rightarrow \underline{g'(t) = f'(\gamma(t)) \cdot \dot{\gamma}(t)}$$

Aufgabe 3

Cauchy-Riemann Gleichung: $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$; $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

$$\text{a) } u(x, y) := \sin(x^2 - y^2) \cosh(2xy)$$

$$\frac{\partial u}{\partial x} = 2x \cdot \cos(x^2 - y^2) \cosh(2xy) + 2y \cdot \sin(x^2 - y^2) \sinh(2xy)$$

$$\frac{\partial u}{\partial y} = -2y \cdot \cos(x^2 - y^2) \cosh(2xy) + 2x \cdot \sin(x^2 - y^2) \sinh(2xy)$$

$$v(x, y) = -\cos(x^2 - y^2) \sinh(2xy)$$

$$\frac{\partial v}{\partial x} = 2x \cdot \sin(x^2 - y^2) \sinh(2xy) - 2y \cos(x^2 - y^2) \cosh(2xy)$$

$$\frac{\partial v}{\partial y} = -2y \cdot \sin(x^2 - y^2) \sinh(2xy) - 2x \cdot \cos(x^2 - y^2) \cosh(2xy)$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \checkmark$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \quad \checkmark$$

\Rightarrow Da die Cauchy-Riemann Gleichungen gelten, ist die Gleichung holomorph

$$b) \quad u(x, y) := e^{x^2-y^2} \cos(2xy)$$

$$\frac{\partial u}{\partial x} = 2x \cdot e^{x^2-y^2} \cdot \cos(2xy) - 2y \cdot e^{x^2-y^2} \cdot \sin(2xy)$$

$$\frac{\partial u}{\partial y} = -2y \cdot e^{x^2-y^2} \cdot \cos(2xy) - 2x \cdot e^{x^2-y^2} \cdot \sin(2xy)$$

$$v(x, y) := e^{x^2-y^2} \sin(2xy)$$

$$\frac{\partial v}{\partial x} = 2x \cdot e^{x^2-y^2} \cdot \sin(2xy) + 2y \cdot e^{x^2-y^2} \cdot \cos(2xy)$$

$$\frac{\partial v}{\partial y} = -2y \cdot e^{x^2-y^2} \cdot \sin(2xy) + 2x \cdot e^{x^2-y^2} \cdot \cos(2xy)$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \checkmark$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \quad \checkmark$$

\Rightarrow Da die Cauchy-Riemann Gleichung gelten, ist die Gleichung holomorph

Aufgabe 4

$$x = r \cdot \sin(\varphi)$$

$$y = r \cdot \cos(\varphi)$$

$$r = \sqrt{x^2 + y^2}$$

$$\varphi = \arctan\left(\frac{x}{y}\right)$$

$$\tilde{u}(x, y) = \tilde{u}(r \cdot \sin(\varphi), r \cdot \cos(\varphi))$$

$$\tilde{v}(x, y) = \tilde{v}(r \cdot \sin(\varphi), r \cdot \cos(\varphi))$$

$$\frac{\partial \tilde{u}}{\partial r} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial r} \quad \parallel \text{ Kettenregel}$$

$$\frac{\partial \tilde{u}}{\partial \varphi} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial \varphi} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial \varphi}$$

$$\frac{\partial \tilde{v}}{\partial r} = \frac{\partial v}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial v}{\partial y} \frac{\partial y}{\partial r}$$

$$\frac{\partial \tilde{v}}{\partial \varphi} = \frac{\partial v}{\partial x} \frac{\partial x}{\partial \varphi} + \frac{\partial v}{\partial y} \frac{\partial y}{\partial \varphi}$$

$$\frac{\partial \tilde{u}}{\partial r} = \frac{\partial u}{\partial x} \cdot \cos(\varphi) + \frac{\partial u}{\partial y} \cdot \sin(\varphi) \Rightarrow \frac{\partial v}{\partial y} \cdot \cos(\varphi) - \frac{\partial v}{\partial x} \cdot \sin(\varphi)$$

$$\frac{\partial \tilde{u}}{\partial \varphi} = \frac{\partial u}{\partial x} \cdot -r \cdot \sin(\varphi) + \frac{\partial u}{\partial y} \cdot r \cdot \cos(\varphi) \Rightarrow \frac{\partial v}{\partial y} \cdot -r \cdot \sin(\varphi) - \frac{\partial v}{\partial x} \cdot r \cdot \cos(\varphi)$$

$$\frac{\partial \tilde{v}}{\partial r} = \frac{\partial v}{\partial x} \cdot \cos(\varphi) + \frac{\partial v}{\partial y} \cdot \sin(\varphi) \Rightarrow -\frac{\partial u}{\partial y} \cdot \cos(\varphi) + \frac{\partial u}{\partial x} \cdot \sin(\varphi)$$

$$\frac{\partial \tilde{v}}{\partial \varphi} = \frac{\partial v}{\partial x} \cdot -r \cdot \sin(\varphi) + \frac{\partial v}{\partial y} \cdot r \cdot \cos(\varphi) \Rightarrow -\frac{\partial u}{\partial y} \cdot -r \cdot \sin(\varphi) + \frac{\partial u}{\partial x} \cdot r \cdot \cos(\varphi)$$

$$r \cdot \frac{\partial \tilde{u}}{\partial r} = r \left(\frac{\partial v}{\partial y} \cdot \cos(\varphi) - \frac{\partial v}{\partial x} \cdot \sin(\varphi) \right) = \frac{\partial v}{\partial y} \cdot r \cdot \cos(\varphi) - \frac{\partial v}{\partial x} \cdot r \cdot \sin(\varphi) = \frac{\partial \tilde{v}}{\partial r} \cdot r$$

$$-r \cdot \frac{\partial \tilde{v}}{\partial \varphi} = -r \left(-\frac{\partial u}{\partial y} \cdot \cos(\varphi) + \frac{\partial u}{\partial x} \cdot \sin(\varphi) \right) = \frac{\partial u}{\partial y} \cdot r \cdot \cos(\varphi) - \frac{\partial u}{\partial x} \cdot r \cdot \sin(\varphi) = \frac{\partial \tilde{u}}{\partial \varphi} \cdot r \quad \square$$

Seite 6

Aufgabe 1

$$\begin{aligned} \text{i)} \quad f(t) &= z_0 + t(z_1 - z_0) \\ &= i + t(4 + 4i) \end{aligned}$$

$$\Rightarrow f(t) = 4t + i(1 + 4t)$$

$$\begin{aligned} \text{ii)} \quad f(t) &= z_0 + r \cdot e^{it} \\ &= 3 - 2i + 5(\cos(t) + i \sin(t)) \end{aligned}$$

$$\Rightarrow f(t) = 3 + 5\cos(t) + i(\sin(t) - 2)$$

Aufgabe 2

$$\begin{aligned} \text{i)} \quad \int_{\gamma} f(z) + g(z) \, dz &= \int_0^1 (f(\gamma(t)) + g(\gamma(t))) \cdot \gamma'(t) \, dt \quad \left\| \text{Summenregel für Integrale:} \right. \\ &= \int_0^1 f(\gamma(t)) \cdot \gamma'(t) \, dt + \int_0^1 g(\gamma(t)) \cdot \gamma'(t) \, dt = \int_{\gamma} f(z) \, dz + \int_{\gamma} g(z) \, dz \quad \square \\ &\quad \left. \int f(x) + g(x) \, dx = \int f(x) \, dx + \int g(x) \, dx \right\} \end{aligned}$$

$$\text{ii)} \quad |\gamma| := \int_0^1 |\gamma'(t)| \, dt$$

$$\left| \int_{\gamma} f(z) \, dz \right| = \left| \int_0^1 f(\gamma(t)) \cdot \gamma'(t) \, dt \right| \leq \int_0^1 |f(\gamma(t)) \cdot \gamma'(t)| \, dt = \int_0^1 |f(\gamma(t))| \cdot |\gamma'(t)| \, dt$$

$$\text{Sei } |f(\gamma(t))| \text{ maximal } (\max_{t \in [0,1]} |f(\gamma(t))|)$$

$$\Rightarrow \max_{t \in [0,1]} |f(\gamma(t))| \cdot \int_0^1 |\gamma'(t)| \, dt = \max_{t \in [0,1]} |f(\gamma(t))| \cdot |\gamma|$$

$$\text{Und es gilt auch } \max_{t \in [0,1]} |f(\gamma(t))| = \max_{z \in \gamma([0,1])} |f(z)|$$

(Beide Aussagen sind äquivalent)

□

$$\begin{aligned} \text{iii)} \quad \overline{\int_{\gamma} f(z) \, dz} &= \overline{\int_0^1 f(e^{2\pi i t}) \cdot 2\pi i e^{2\pi i t} \, dt} = \int_0^1 \overline{f(e^{2\pi i t}) \cdot 2\pi i e^{2\pi i t}} \, dt \\ &= \int_0^1 \overline{f(e^{2\pi i t})} \cdot \overline{2\pi i e^{2\pi i t}} \, dt \quad \left\| \begin{aligned} \overline{2\pi i} &= -2\pi i \text{ und } \overline{e^{2\pi i t}} = \cos(2\pi t) + i\sin(2\pi t) \\ &= \cos(2\pi t) - i\sin(2\pi t) = \cos(-2\pi t) + i\sin(-2\pi t) \end{aligned} \right. \\ &= - \int_0^1 \overline{f(z)} \cdot (2\pi i e^{-2\pi i t}) \, dt \quad \left\| e^{-2\pi i t} = \frac{e^{2\pi i t}}{e^{4\pi i t}} \right. \\ &= - \int_0^1 \frac{\overline{f(z)}}{z^2} \cdot 2\pi i e^{2\pi i t} \, dt = \int_{\gamma} \frac{\overline{f(z)}}{z^2} \, dz \quad \square \end{aligned}$$

Aufgabe 3

$$\int_{\gamma} \operatorname{Im}(z) dz = \int_0^{2\pi} \operatorname{Im}(z) z'(t) dt = \int_0^{2\pi} \sin(t) \cdot (-\sin(t) + i \cos(t)) dt = \int_0^{2\pi} -\sin(t)^2 + i \sin(t) \cos(t) dt$$

$$\frac{dz}{dt} = z'(t)$$

$$\Rightarrow \int_0^{2\pi} -\sin(t)^2 dt = - \int_0^{2\pi} \frac{1 - \cos(2t)}{2} dt = - \left[\frac{t}{2} - \frac{\sin(2t)}{4} \right]_0^{2\pi} = -\pi$$

$$\Rightarrow \int_0^{2\pi} i \cdot \sin(t) \cdot \cos(t) dt = i \int_0^{2\pi} \frac{1}{2} \cdot \sin(2t) dt = \frac{i}{2} \left[-\frac{\cos(2t)}{2} \right]_0^{2\pi} = 0$$

$$\Rightarrow -\pi + 0 = -\pi$$

Aufgabe 4

$$\int_{\gamma} \frac{1}{z^n} dz \quad \left\| \quad z(t) = \cos(t) - i \sin(t) = e^{-it} \text{ (Uhrzeigersinn)} \right.$$

$$= \int_0^{2\pi} \frac{1}{z^n} \cdot z'(t) dt = \int_0^{2\pi} e^{int} \cdot -i \cdot e^{-it} dt = \int_0^{2\pi} -i \cdot e^{i(n-1)t} dt = \int_0^{2\pi} -i \cdot e^{(n-1)it} dt$$

$$\Rightarrow n=1$$

$$\rightarrow \int_0^{2\pi} -i dt = [-it]_0^{2\pi} = -i2\pi$$

$$\Rightarrow n \neq 1$$

$$\rightarrow \int_0^{2\pi} -i \cdot e^{(n-1)it} dt = -i \left[\frac{e^{(n-1)it}}{(n-1)i} \right]_0^{2\pi} = 1 - 1 = 0 \quad \left\| \quad e^{i2\pi(n-1)} = \cos((n-1)2\pi) + i \sin((n-1)2\pi) \right.$$

$$\Rightarrow i2\pi - 0 = -i2\pi$$

DISCLAIMER

Die Notizen zu den Stack Aufgaben sind auf meine Werte angepasst. Die Werte können von deinen Aufgaben abweichen.

Seite 1

Frage 1

$$(8-i)(5i+3) = 40i + 24 + 5 - 3i = 37i + 29$$

$$i(5i+3) = -5 + 3i$$

$$\bar{w} = -5i + 3$$

$$\frac{8-i}{5i+3} = \frac{(8-i)(5i-3)}{-34} = \frac{40i - 24 + 5 + 3i}{-34} = -\frac{43i}{34} + \frac{19}{34}$$

$$|z| \sqrt{8^2 + 1^2} = \sqrt{65}$$

Frage 2

$$\cos(-\frac{\pi}{4}) > 0; \cos(\frac{\pi}{4}) > 0 \Rightarrow \operatorname{Re}(z) > 0$$

$$\sin(-\frac{\pi}{4}) < 0; \sin(\frac{\pi}{4}) > 0 \Rightarrow \operatorname{Im}(z) > / = / < 0$$

Frage 3

$$(4+2i)(4+2i) = 16 + 8i + 8i - 4 = 12 + 16i$$

$$(12+16i)(4+2i) = 48 + 24i + 64i - 32 = 16 + 88i$$

Bemerkung: Teilweise ist es einfacher potenzierte komplexe Gleichungen
in Polarform zu berechnen.

Serie 2

Serie 3

Frage 1

$\operatorname{Im}(z) = 0$ || Der Im von z muss 0 sein damit $z \in \mathbb{R}$

$$z = e^{\frac{3\pi}{4}i} \cdot (\sqrt{2} + bi)$$

$$= \left(\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right) \cdot (\sqrt{2} + bi)$$

$$= \sqrt{2} \cos\left(\frac{3\pi}{4}\right) + bi \cos\left(\frac{3\pi}{4}\right) + \sqrt{2} i \sin\left(\frac{3\pi}{4}\right) - b \sin\left(\frac{3\pi}{4}\right) \quad || \operatorname{Im}(z) = 0$$

$$\Rightarrow bi \cos\left(\frac{3\pi}{4}\right) + \sqrt{2} i \sin\left(\frac{3\pi}{4}\right) = 0$$

$$bi \cos\left(\frac{3\pi}{4}\right) = -\sqrt{2} i \sin\left(\frac{3\pi}{4}\right)$$

$$\cancel{bi} \cdot \frac{\sqrt{2}}{2} = \cancel{-\sqrt{2}i}$$

$$b = \frac{2\sqrt{2}}{2}$$

Frage 2

$$c_n = \frac{(4n)!}{(-2-i)^n \cdot (n!)^4}$$

$$\rho = \frac{1}{\limsup_{n \rightarrow \infty} \sqrt[n]{|c_n|}}$$

$$\limsup_{n \rightarrow \infty} \sqrt[n]{\left| \frac{(4n)!}{(-2-i)^n \cdot (n!)^4} \right|} = \limsup_{n \rightarrow \infty} \sqrt[n]{\left| \frac{1}{(-2-i)^n} \cdot \frac{(4n)!}{(n!)^4} \right|}$$

$$(4n)! = (4n)(4n-1)(4n-2) \dots 4 \cdot 3 \cdot 2 \cdot 1$$

Wenn wir die Multiplikation in 4er Gruppen aufteilen:

$$\left. \begin{array}{l} [(4n)(4n-1)(4n-2)(4n-3)] \leq (4n)^4 \\ \vdots \\ [4 \cdot 3 \cdot 2 \cdot 1] \leq (4n)^4 \end{array} \right\} n\text{-Mal}$$

$$\Rightarrow (4n)! \leq (4n)^4 \cdot (4n)^4 \cdot \dots = (4n)^{4n}$$

$$(n!)^4 = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$$

Wenn wir die Multiplikation in 4er Gruppen aufteilen

$$\left. \begin{array}{l} [n \cdot (n-1)(n-2)(n-3)] \leq n^4 \\ \vdots \\ [4 \cdot 3 \cdot 2 \cdot 1] \leq n^4 \end{array} \right\} n\text{-Mal}$$

$$\Rightarrow (n!)^4 \leq n^{4n}$$

Frage 2

$$\lim_{n \rightarrow \infty} \sup \sqrt[n]{\left| \frac{1}{\sqrt{5}} \left| \frac{(4n)!}{(n!)^4} \right| \right|} \leq \sqrt[n]{\left| \frac{1}{\sqrt{5}} \left| \frac{(4n)^{4n}}{(n)^{4n}} \right| \right|} = \sqrt[n]{\left| \frac{1}{\sqrt{5}} \left| \frac{4^{4n} n^{4n}}{n^{4n}} \right| \right|} = \frac{256}{\sqrt{5}}$$

$$R = \frac{1}{\frac{256}{\sqrt{5}}} = \frac{\sqrt{5}}{256}$$

Frage 3

$$z = (-\sqrt{3} + i)^7 \quad || \quad z = r^n \cdot e^{in\phi}$$

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = 2$$

$$\phi = \arctan\left(\frac{1}{-\sqrt{3}}\right) = -\frac{\pi}{6}$$

$$\Rightarrow z = 2^7 \cdot e^{i\frac{7\pi}{6}} = 128 e^{-i\frac{7\pi}{6}}$$

Serie 4

Frage 1

$$z^5 = -\frac{3}{2} + \frac{3^{\frac{3}{2}}}{2}i$$

$$r = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3} = 3$$

$$\phi = \arctan\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{3}{2}}\right) + \pi = \frac{2\pi}{3} + 2\pi k$$

$$s_k = \frac{2\pi}{3} + 2\pi k$$

$$\varphi = \frac{2\pi}{15} + \frac{2\pi k}{5} \quad \parallel \text{ mit } 0 \leq k \leq 4$$

$$z_1 = 3^{\frac{1}{5}} \cdot e^{i \cdot \left(\frac{2\pi}{15}\right)}$$

$$z_2 = 3^{\frac{1}{5}} \cdot e^{i \cdot \left(\frac{2\pi}{15} + \frac{2\pi}{5}\right)}$$

$$z_3 = 3^{\frac{1}{5}} \cdot e^{i \cdot \left(\frac{2\pi}{15} + \frac{4\pi}{5}\right)}$$

$$z_4 = 3^{\frac{1}{5}} \cdot e^{i \cdot \left(\frac{2\pi}{15} + \frac{6\pi}{5}\right)}$$

$$z_5 = 3^{\frac{1}{5}} \cdot e^{i \cdot \left(\frac{2\pi}{15} + \frac{8\pi}{5}\right)}$$

Frage 2

$$\text{Log}\left(2^{\frac{3}{2}} + 2^{\frac{3}{2}}i\right) \parallel \text{Log}(z) = \log(|z|) + \text{Arg}(z) \cdot i$$

$$|z| = \sqrt{\left(2^{\frac{3}{2}}\right)^2 + \left(2^{\frac{3}{2}}\right)^2} = \sqrt{2^3 + 2^3} = \sqrt{16} = 4$$

$$\text{Arg}(z) = \frac{\pi}{4}$$

$$\Rightarrow \log(4) + \frac{\pi}{4}i$$

Frage 1

$$u(x,y) = 4x^3y - 4xy^3$$

$$\frac{\partial u}{\partial x} = 12x^2y - 4y^3$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = 24xy$$

$$\frac{\partial u}{\partial y} = 4x^3 - 12xy^2$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = -24xy$$

} = 0

$$v(x,y) = y^4 - 6x^2y^2 + x^4$$

$$\frac{\partial v}{\partial x} = -12xy^2 + 4x^3$$

$$\Rightarrow \frac{\partial^2 v}{\partial x^2} = 12x^2$$

$$\frac{\partial v}{\partial y} = 4y^3 - 12x^2y$$

$$\Rightarrow \frac{\partial^2 v}{\partial y^2} = 12y^2$$

} \neq 0

 \Rightarrow Nicht holomorph

Frage 2

$$f(z) = -z^4 \quad || z = x+iy$$

$$= -(x+iy)^4 = -(x^4 + 4x^3iy - 6x^2y^2 - 4xy^3 + y^4)$$

$$= -x^4 - 4x^3iy + 6x^2y^2 + 4xy^3 - y^4$$

$$u(x,y) = -x^4 + 6x^2y^2 - y^4; \quad v(x,y) = -4x^3y + 4xy^3$$

$$\frac{\partial u}{\partial x} = -4x^3 + 12xy^2$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = -12x^2 + 12y^2$$

$$\frac{\partial u}{\partial y} = 12x^2y - 4y^3$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = 12x^2 - 12y^2$$

$$\frac{\partial v}{\partial x} = -12x^2y + 4y^3$$

$$\Rightarrow \frac{\partial^2 v}{\partial x^2} = -24xy$$

$$\frac{\partial v}{\partial y} = -4x^3 + 12xy^2$$

$$\Rightarrow \frac{\partial^2 v}{\partial y^2} = 24xy$$

 \Rightarrow holomorph

Frage 3

$$\lim_{z \rightarrow 0} \left(\frac{z}{\bar{z}} \right)^2 \quad || z = x+iy$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \left(\frac{(x+iy)}{(x-iy)} \right)^2 = \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^2 + 2xy - y^2}{x^2 - 2xy - y^2} \quad || \text{Limes von } x \text{ und } y \text{ separat betrachten}$$

$$\lim_{\substack{x \rightarrow 0 \\ y=0}} \frac{x^2}{x^2} = 1^2 = 1$$

$$\lim_{\substack{y \rightarrow 0 \\ x=0}} \frac{-y^2}{-y^2} = 1^2 = 1$$

Kontrolle für $x=y \Rightarrow$ da es für alle Richtungen gelten muss.

$$z = x + ix$$

$$\lim_{x \rightarrow \infty} \left(\frac{x+ix}{x-ix} \right)^2 = \lim_{x \rightarrow \infty} \frac{\cancel{x} + 2ix^2 - \cancel{x}}{\cancel{x}^2 - 2ix^2 - \cancel{x}} = -1$$

\Rightarrow Die Limes sind nicht gleich. Somit existiert der Limes nicht

Frage 5

$$f(z) = \frac{z-4}{z-3} = \frac{x+iy-4}{x+iy-3} = \frac{(x-4)+iy}{(x-3)+iy} \parallel \frac{(x-3)-iy}{(x-3)-iy}$$
$$= \frac{(x-4)(x-3) - iy(x-4) + iy(x-3) + y^2}{(x-3)^2 + y^2}$$

$$\Rightarrow \operatorname{Re}(f) = \frac{(x-4)(x-3) + y^2}{(x-3)^2 + y^2} ; \operatorname{Im}(f) = \frac{-y(x-4) + y(x-3)}{(x-3)^2 + y^2}$$

Serie 6

Frage 1

$$z = t^2$$

$$\frac{dz}{dt} = 2t$$

$$dz = 2t dt$$

$$\int_0^2 (t^2)^2 \cdot 2t dt = \int_0^2 t^4 \cdot 2t dt = 2 \int_0^2 t^5 dt = 2 \left[\frac{t^6}{6} \right]_0^2 = \frac{2 \cdot 64}{6} = \frac{128}{6} = \frac{64}{3}$$

Frage 2

$$z_1(t) = 2i + 3 + t(2i + 1 - 2i - 3) = 2i - 2t + 3$$

$$dz = -2 dt$$

$$z_2(t) = 2i + 1 + t(1 - 2i - 1) = 2i - 2it - 1$$

$$dz = -2i dt$$

$$z_3(t) = 1 + t(3 - 1) = 1 + 2t$$

$$dz = 2 dt$$

$$z_4(t) = 3 + t(2i + 3 - 3) = 3 + 2it$$

$$dz = 2i dt$$

$$|z_1|^2 = (3 - 2t)^2 + 2^2 = 13 - 12t + 4t^2$$

$$|z_2|^2 = (2 - 2t)^2 + 1^2 = 5 - 8t + 4t^2$$

$$|z_3|^2 = (1 + 2t)^2 = 1 + 2t + 4t^2$$

$$|z_4|^2 = 2t^2 + 3^2 = 9 + 4t^2$$

$$\int_0^1 (13 - 12t + 4t^2) (-2) dt = -2 \int_0^1 (13 - 12t + 4t^2) dt = -2 \left[\frac{4t^3}{3} - 6t^2 + 13t \right]_0^1$$

$$= -2 \left[\frac{4}{3} - 6 + 13 \right] = -\frac{50}{3}$$

$$\int_0^1 (5 - 8t + 4t^2) (-2i) dt = -2i \int_0^1 (5 - 8t + 4t^2) dt = -2i \left[\frac{4t^3}{3} - 4t^2 + 5t \right]_0^1$$

$$= -2i \left[\frac{4}{3} - 4 + 5 \right] = -\frac{14}{3}i$$

$$\int_0^1 (1+2t+4t^2)(2) dt = 2 \cdot \int_0^1 1+2t+4t^2 dt = 2 \cdot \left[\frac{4t^3}{3} + t^2 + t \right]_0^1$$

$$= 2 \cdot \left[\frac{4}{3} + 1 + 1 \right] = \frac{26}{3}$$

$$\int_0^1 (3+4t^2)(2i) dt = 2i \int_0^1 3+4t^2 dt = 2i \left[\frac{4t^3}{3} + 3t \right]_0^1$$

$$2i \left[\frac{4}{3} + 3 \right] = \frac{62}{3} i$$

$$\Rightarrow \int_{\Gamma} |z|^2 dz = -\frac{24}{3} + \frac{48}{3} i = -8 + 16i$$

Frage 3

$$z(t) = 3+bi + t(5+4i - 3-bi) = 3+bi + 2t - 2it$$

$$dz = 2 - 2i dt$$

$$\int_0^1 (3-bi + 2t + 2it)(2-2i) dt = \int_0^1 6 - 12i + 4t + 4it - bi - 12i - 4ti + 4t dt$$

$$= \int_0^1 -b + 8t - 16i dt = \left[-bt + 4t^2 - 16it \right]_0^1 = -2 - 16i$$

Frage 4

$$z(t) = \frac{\pi}{2} e^{it}$$

$$dz = \frac{\pi}{2} i e^{it} dt$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\pi}{2} e^{it} \right)^3 \left(\frac{\pi}{2} i e^{it} \right) dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\pi^3}{8} e^{3it} \frac{\pi}{2} i e^{it} dt = \frac{\pi^4}{16} i \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{4it} dt$$

$$= \frac{\pi^4}{16} i \left[\frac{e^{4it}}{4i} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 1 - 1 = 0$$

Frage 5

$$\lim_{z \rightarrow z_0} \frac{f(z) - f(z_0)}{z - z_0} = \lim_{z \rightarrow z_0} \frac{(z+3)^2 - (z_0+3)^2}{z - z_0} = \lim_{z \rightarrow z_0} \frac{z^2 + 6z + 9 - z_0^2 - 6z_0 - 9}{z - z_0}$$

$$= \lim_{z \rightarrow z_0} \frac{(z - z_0)^2 + 6(z - z_0)}{z - z_0} = \lim_{z \rightarrow z_0} \frac{(z + z_0)(z - z_0) + 6(z - z_0)}{z - z_0} = 2z_0 + 6$$

$$\begin{aligned} f(x,y) &= ((x+iy)+3)^2 = (x+3)^2 + 2iy(x+3) - y^2 = x^2 + 6x + 9 + 2ixy + 6iy - y^2 \\ &= (x^2 - y^2 + 6x + 9) + i(2xy + 6y) \end{aligned}$$

$$\frac{\partial f_1}{\partial x} = 2x + b + 2yi$$

$$\frac{\partial f_1}{\partial y} = -2y + 2xi + bi$$

$$u(x,y) = x^2 - y^2 + 6x + 9$$

$$v(x,y) = 2xy + 6y$$

$$\frac{\partial u}{\partial x} = 2x + 6$$

$$\frac{\partial u}{\partial y} = -2y$$

$$\frac{\partial v}{\partial x} = 2y$$

$$\frac{\partial v}{\partial y} = 2x + 6$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \checkmark$$

\Rightarrow Es gilt die Cauchy-Riemann Gleichung

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} \quad \checkmark$$