

DISCLAIMER

Ich übernehme keine Haftung über mögliche Fehler in den Notizen. Es hat sicherlich ein paar drinnen.

Fehler können per Mail an jirruh@ethz.ch gemeldet werden.

Serie 1

Aufgabe 1

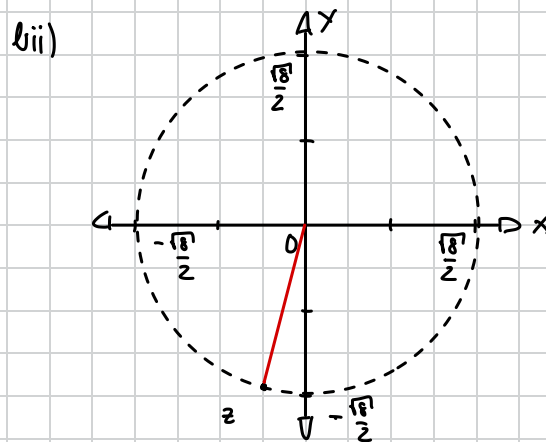
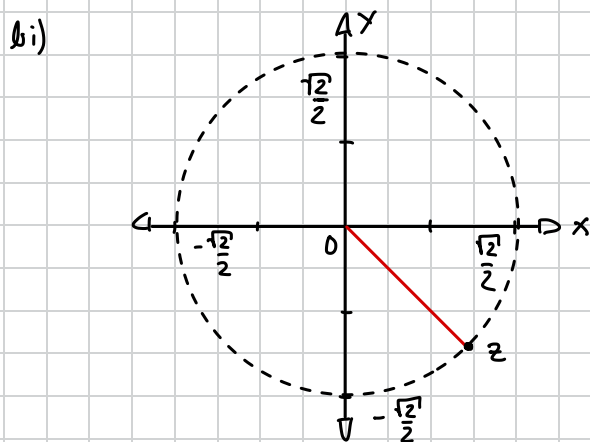
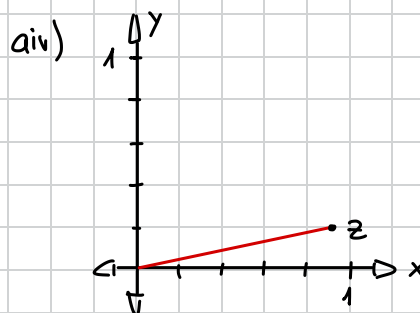
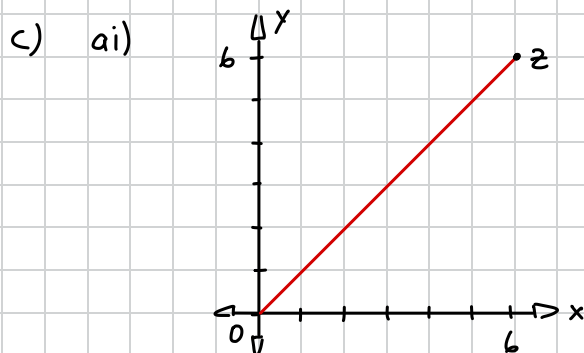
a) i) $6 + 6i$ ii) $-10 + i$ iii) $3 - 39i$

iv) $\frac{(12 + 3i)}{(12 + 6i)} = \frac{(12 + 3i)(12 - 6i)}{144 + 36} = \frac{162 - 36i}{180}$

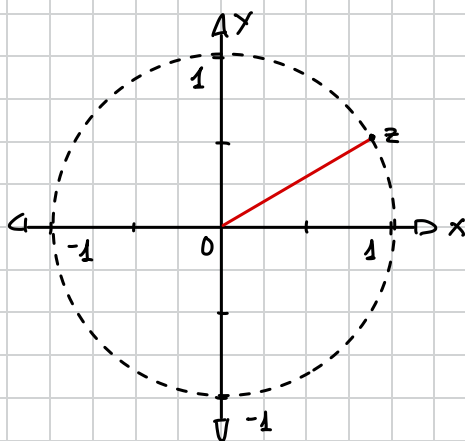
b) i) $-i \Rightarrow 1(\cos(-\frac{\pi}{2}) + i \sin(-\frac{\pi}{2}))$
 $1 - i \Rightarrow \sqrt{2}(\cos(\frac{\pi}{4}) + i \sin(-\frac{\pi}{4}))$ } $|z| = \sqrt[4]{2} = \frac{\sqrt{2}}{2}$
 $\arg(z) = (-\frac{\pi}{2} + \frac{\pi}{4}) = -\frac{\pi}{4}$
 $\Rightarrow z = \frac{\sqrt{2}}{2}(\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4}))$

ii) $-2 - 2i \Rightarrow \sqrt{8}(\cos(-\frac{3\pi}{4}) + i \sin(-\frac{\pi}{4}))$
 $1 + \sqrt{3}i \Rightarrow 2(\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))$ } $|z| = \sqrt[8]{2}$
 $\arg(z) = (-\frac{3\pi}{4} - \frac{\pi}{3}) = -\frac{13\pi}{12} = \frac{11\pi}{12}$
 $\Rightarrow z = \frac{\sqrt{8}}{2}(\cos(\frac{11\pi}{12}) + i \sin(\frac{11\pi}{12}))$

iii) $1 + \sqrt{3}i \Rightarrow 2(\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))$
 $\sqrt{3} + i \Rightarrow 2(\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6}))$ } $|z| = 1$
 $\arg(z) = (\frac{\pi}{3} - \frac{\pi}{6}) = \frac{\pi}{6}$
 $z = 1(\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6}))$

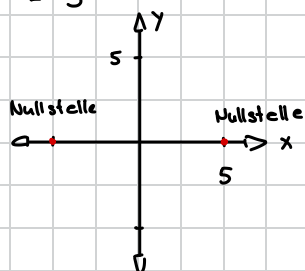


biii)



Aufgabe 2

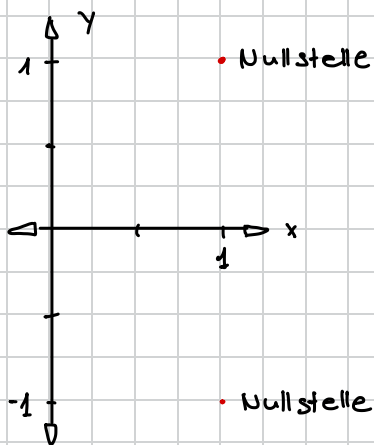
i) $z = \pm 5$



ii) $z^2 - 2z + 2 = 0$

$$z_{1,2} = 1 \pm \sqrt{1 - 2}$$

$$= 1 \pm i$$



iii) $z^3 + z^2 - 2 : z - 1 = z^2 + 2z + 2$

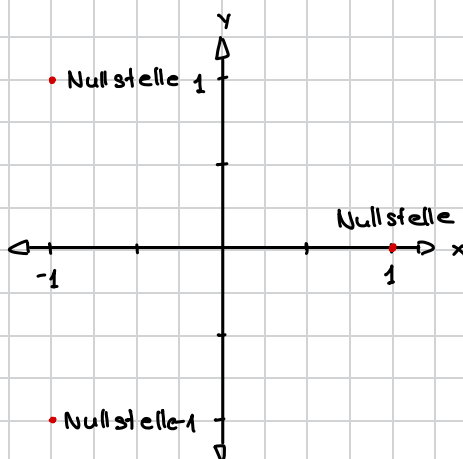
$$\begin{array}{r} z^3 + z^2 - 2 \\ \underline{z^3 - z^2} \\ 2z^2 - 2z \\ \underline{2z^2 - 2z} \\ 2z - 2 \end{array}$$

$$z_1 = 1$$

$$z^2 + 2z + 2 = 0$$

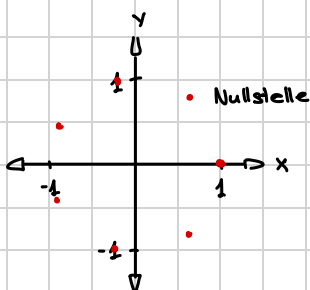
$$z_{2,3} = -1 \pm \sqrt{1 - 2}$$

$$= -1 \pm i$$



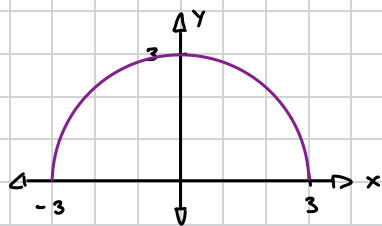
iv) $z = 1 (\cos(\varphi) + i \cdot \sin(\varphi))$

$$7\varphi = 2\pi n \Rightarrow \varphi = \frac{2\pi n}{7}$$



Aufgabe 1

i) $M = \{z \in \mathbb{C} \mid |z| = 3, \operatorname{Im}(z) \geq 0\}$



ii) $M = \left\{ z \in \mathbb{C} \mid \frac{|z+2-2i|}{|z+i|} = 2 \right\}$
 $= \left\{ z \in \mathbb{C} \mid \frac{|x+2+i(y-2)|}{|x+i(y+1)|} = 2 \right\}$

$$(|x+2+i(y-2)|)^2 = (2|x+i(y+1)|)^2$$

$$(x+2)^2 + (y-2)^2 = 4(x^2 + (y+1)^2)$$

$$x^2 + 4x + 4 + y^2 - 4y + 4 = 4x^2 + 4y^2 + 8y + 4$$

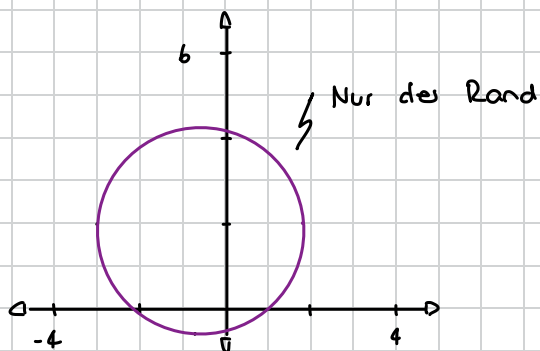
$$-3x^2 + 4x + 4 = 3y^2 + 12y$$

$$3x^2 - 4x + 3y^2 + 12y = 4$$

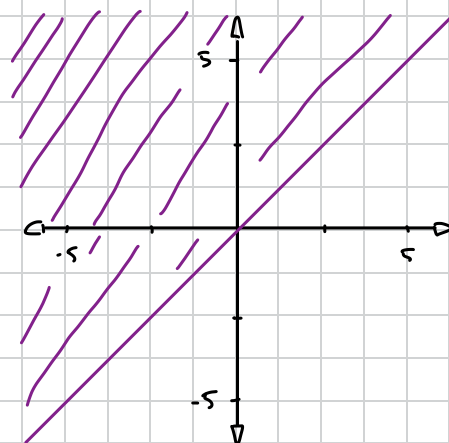
$$x^2 - \frac{4}{3}x + y^2 + 4y = \frac{4}{3}$$

$$\left(x - \frac{2}{3}\right)^2 + (y+2)^2 = \frac{52}{3}$$

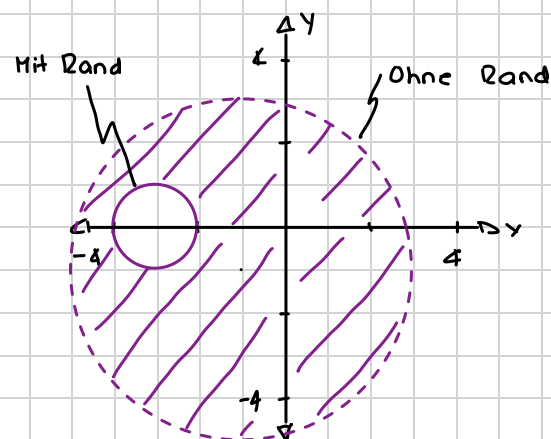
$$M = \left\{ z \in \mathbb{C} \mid \left| z - \frac{2}{3} + 2i \right| = \frac{2\sqrt{13}}{3} \right\}$$



iii) $M = \{z \in \mathbb{C} \mid \operatorname{Im}(z) \geq \operatorname{Re}(z)\}$



iv) $M = \{z \in \mathbb{C} \mid |z-3| \geq 1 \text{ und } |z-1-i| < 4\}$



DISCLAIMER

Die Notizen zu den Stack Aufgaben sind auf meine Werte angepasst. Die Werte können von deinen Aufgaben abweichen.

Seite 1

Frage 1

$$(8-i)(5i+3) = 40i + 24 + 5 - 3i = 37i + 29$$

$$i(5i+3) = -5 + 3i$$

$$\bar{w} = -5i + 3$$

$$\frac{8-i}{5i+3} = \frac{(8-i)(5i-3)}{-34} = \frac{40i - 24 + 5 + 3i}{-34} = -\frac{43i}{34} + \frac{19}{34}$$

$$|z| \sqrt{8^2 + 1^2} = \sqrt{65}$$

Frage 2

$$\cos\left(-\frac{\pi}{4}\right) > 0; \cos\left(\frac{\pi}{4}\right) > 0 \Rightarrow \operatorname{Re}(z) > 0$$

$$\sin\left(-\frac{\pi}{4}\right) < 0; \sin\left(\frac{\pi}{4}\right) > 0 \Rightarrow \operatorname{Im}(z) > 0 \text{ oder } < 0$$

Frage 3

$$(4+2i)(4+2i) = 16 + 8i + 8i - 4 = 12 + 16i$$

$$(12+16i)(4+2i) = 48 + 24i + 64i - 32 = 16 + 88i$$

Bemerkung: Teilweise ist es einfacher potenzierte komplexe Gleichungen
in Polarform zu berechnen.

Serie 2

Die Stackaufgaben für Serie 2 waren sehr einfach. Bei Fragen kann man mich kontaktieren.