

DISCLAIMER

Ich übernehme keine Haftung über mögliche Fehler in den Notizen. Es hat sicherlich ein paar drinnen.

Fehler können per Mail an jirruh@ethz.ch gemeldet werden.

Serie 1

Aufgabe 1

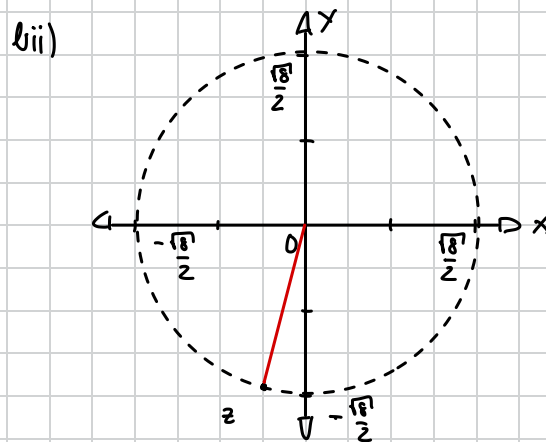
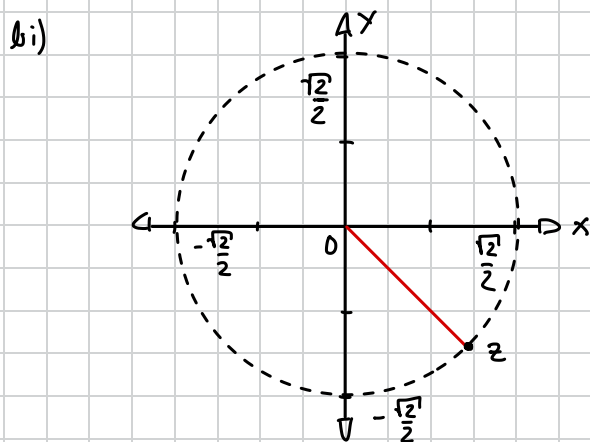
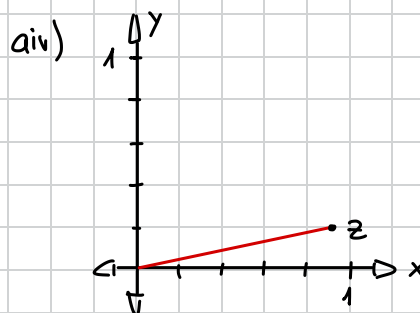
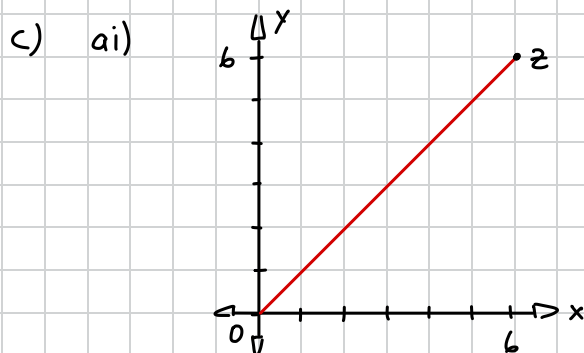
a) i) $6 + 6i$ ii) $-10 + i$ iii) $3 - 39i$

iv) $\frac{(12 + 3i)}{(12 + 6i)} = \frac{(12 + 3i)(12 - 6i)}{144 + 36} = \frac{162 - 36i}{180}$

b) i) $-i \Rightarrow 1(\cos(-\frac{\pi}{2}) + i \sin(-\frac{\pi}{2}))$
 $1 - i \Rightarrow \sqrt{2}(\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4}))$ } $|z| = \sqrt[4]{2} = \frac{\sqrt{2}}{2}$
 $\arg(z) = (-\frac{\pi}{2} + \frac{\pi}{4}) = -\frac{\pi}{4}$
 $\Rightarrow z = \frac{\sqrt{2}}{2}(\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4}))$

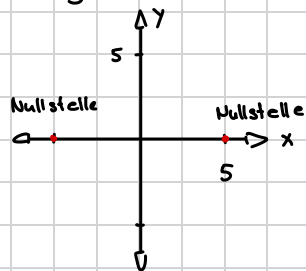
ii) $-2 - 2i \Rightarrow \sqrt{8}(\cos(-\frac{3\pi}{4}) + i \sin(-\frac{3\pi}{4}))$
 $1 + \sqrt{3}i \Rightarrow 2(\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))$ } $|z| = \sqrt[8]{2}$
 $\arg(z) = (-\frac{3\pi}{4} - \frac{\pi}{3}) = -\frac{13\pi}{12} = \frac{11\pi}{12}$
 $\Rightarrow z = \frac{\sqrt{8}}{2}(\cos(\frac{11\pi}{12}) + i \sin(\frac{11\pi}{12}))$

iii) $1 + \sqrt{3}i \Rightarrow 2(\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))$
 $\sqrt{3} + i \Rightarrow 2(\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6}))$ } $|z| = 1$
 $\arg(z) = (\frac{\pi}{3} - \frac{\pi}{6}) = \frac{\pi}{6}$
 $z = 1(\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6}))$



A diagram of the complex plane with a dashed unit circle centered at the origin. The horizontal axis is labeled x and the vertical axis is labeled y . The origin is marked with 0 . The x -axis has tick marks at -1 and 1 . The y -axis has tick marks at 1 and -1 . A point z is marked on the unit circle in the first quadrant, and a red line segment connects the origin to z .

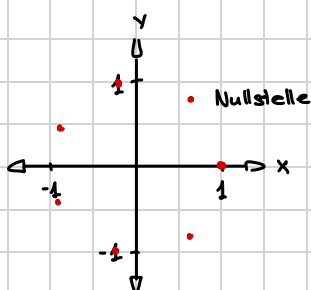
i) $z = \pm 5$



$$z_{1,2} = 1 \pm \sqrt{1 - 2}$$

A hand-drawn coordinate system on a grid. The horizontal axis is labeled x and the vertical axis is labeled y . Both axes have tick marks at 1 and -1 . Two red dots are plotted: one at $(1, 0)$ and one at $(0, 1)$. Each red dot is labeled "Nullstelle" in red text.

$$Tf = 2\pi i n \Rightarrow f = \frac{2\pi i n}{T}$$



$$z^3 - z^2$$

/	$2z^2$
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$$2z^2 - 2z$$

$$/ 27^{-2}$$

$2x - 2$

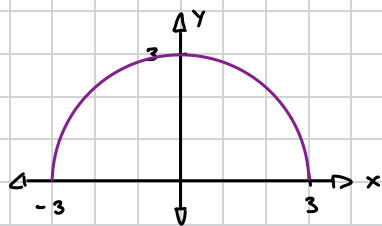
$$z^2 + 2z + 2 = 0$$

$$z_{2,3} = -1 \pm \sqrt{1 - 2}$$

A hand-drawn coordinate system on a grid. The x-axis and y-axis are shown with arrows at their ends. The origin is labeled '0'. On the x-axis, there are tick marks at -1 and 1. On the y-axis, there are tick marks at 1 and -1. Two red dots are plotted on the x-axis at the coordinates (-1, 0) and (1, 0). Next to the dot at (-1, 0), the text 'Nullstelle 1' is written. Next to the dot at (1, 0), the text 'Nullstelle' is written.

Aufgabe 1

i) $M = \{z \in \mathbb{C} \mid |z| = 3, \operatorname{Im}(z) \geq 0\}$



ii) $M = \left\{ z \in \mathbb{C} \mid \frac{|z+2-2i|}{|z+i|} = 2 \right\}$
 $= \left\{ z \in \mathbb{C} \mid \frac{|x+2+i(y-2)|}{|x+i(y+1)|} = 2 \right\}$

$$(|x+2+i(y-2)|)^2 = (2|x+i(y+1)|)^2$$

$$(x+2)^2 + (y-2)^2 = 4(x^2 + (y+1)^2)$$

$$x^2 + 4x + 4 + y^2 - 4y + 4 = 4x^2 + 4y^2 + 8y + 4$$

$$-3x^2 + 4x + 4 = 3y^2 + 12y$$

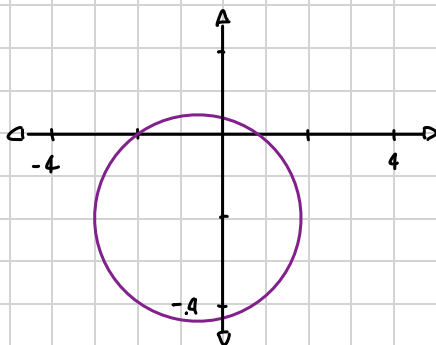
$$3x^2 - 4x + 3y^2 + 12y = 4$$

$$x^2 - \frac{4}{3}x + y^2 + 4y = \frac{4}{3}$$

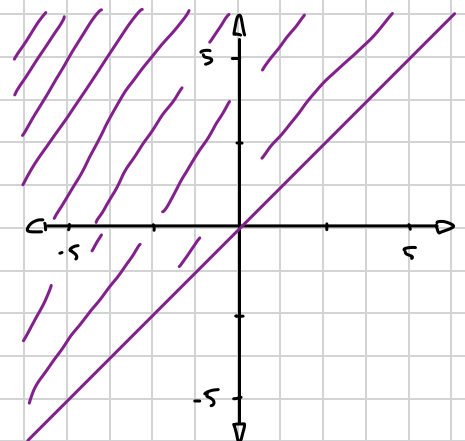
$$\left(x - \frac{2}{3}\right)^2 + (y+2)^2 = \frac{52}{3}$$

$$M = \left\{ z \in \mathbb{C} \mid \left| z - \frac{2}{3} + 2i \right| = \frac{2\sqrt{13}}{3} \right\}$$

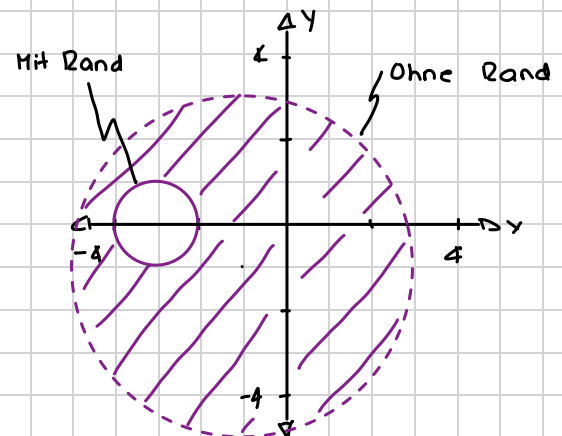
⚡ Nur der Rand



iii) $M = \{z \in \mathbb{C} \mid \operatorname{Im}(z) \geq \operatorname{Re}(z)\}$



iv) $M = \{z \in \mathbb{C} \mid |z-3i| \geq 1 \text{ und } |z-1-i| < 4\}$



Serie 3

Aufgabe 1

a) i) $\lim_{n \rightarrow \infty} \cos(in) = \lim_{n \rightarrow \infty} \cosh(n) = \underline{\underline{\infty}}$

ii) $\lim_{n \rightarrow \infty} 1 + (-1)^n \cdot \frac{1}{n} = 1 + \lim_{n \rightarrow \infty} (-1^n \cdot \frac{1}{n}) = 1 + 0i = \underline{\underline{1}}$ || Wenn $n \rightarrow \infty$ wird $\frac{1}{n} \rightarrow 0$

iii) $\lim_{n \rightarrow \infty} \frac{(n + 2\pi i)^n}{n^n} = \lim_{n \rightarrow \infty} \frac{n^n}{n^n} = \underline{\underline{1}}$ || $2\pi i$ hat kein Einfluss aufs Ergebnis wenn $n \rightarrow \infty$

iv) $\lim_{n \rightarrow \infty} \text{Arg}(1 + (-1)^n \frac{1}{n})$ || $\lim_{n \rightarrow \infty} 1 + (-1)^n \frac{1}{n} = 1$
 $\text{Arg}(1) = \underline{\underline{0}}$

b) $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} + \frac{(\pi i)^n}{n!} \right) = \sum_{n=0}^{\infty} \frac{1}{2^n} + \sum_{n=0}^{\infty} \frac{(\pi i)^n}{n!} = 2 + \exp(\pi i) = \underline{\underline{2 + e^{\pi i}}}$

Aufgabe 2

$$\exp(it) = \cos(t) + i \cdot \sin(t)$$

$$\exp(z_1 + z_2) = \exp(z_1) \cdot \exp(z_2) \quad || \quad z_1 = ix ; z_2 = iy$$

$$\exp(i(x+y)) = \exp(ix) \cdot \exp(iy) \quad || \text{Eulerformel}$$

$$\exp(i(x+y)) = (\cos(x) + i \sin(x)) \cdot (\cos(y) + i \sin(y))$$

$$\cos(x+y) + i \sin(x+y) = (\cos(x) + i \sin(x)) \cdot (\cos(y) + i \sin(y))$$

$$\cos(x+y) + i \sin(x+y) = \cos(x)\cos(y) + \cos(x)i\sin(y) + i\sin(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(x+y) + i \sin(x+y) = (\cos(x)\cos(y) - \sin(x)\sin(y)) + i(\cos(x)\sin(y) + \cos(y)\sin(x))$$

$$\Rightarrow \cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\Rightarrow \sin(x+y) = \cos(x)\sin(y) + \cos(y)\sin(x)$$

|| Ich bin zu faul um es für $x-iy$ zu beweisen :-)

□

Aufgabe 3

i) $\lim_{z \rightarrow 0} \frac{\bar{z} + z^2}{z}$ || $z = x + iy$

$$\lim_{z \rightarrow 0} \frac{x - iy + x^2 + 2xyi - y^2}{x + iy} \quad || \text{Wir müssen den Grenzwert von Re und Im separat betrachten}$$

$$\lim_{y \rightarrow 0} \frac{x - iy + x^2 + 2xyi - y^2}{x + iy} \quad || x=0$$

$$\lim_{y \rightarrow 0} \frac{-iy - y^2}{iy} = \lim_{y \rightarrow 0} -1 - \frac{y^2}{iy} = \lim_{y \rightarrow 0} -1 - \frac{y}{i} = \underline{\underline{-1}}$$

Aufgabe 3

$$\lim_{x \rightarrow 0} \frac{x - iy + x^2 + 2xy - y^2}{x + iy} \quad \parallel y=0$$

$$\lim_{x \rightarrow 0} \frac{x + x^2}{x} = \lim_{x \rightarrow 0} 1 + x = 1$$

$$z_0 = 1 - i \neq 0 \quad \parallel \text{Somit existiert der Limes nicht.}$$

$$i) \quad \lim_{z \rightarrow 0} \frac{\cos(z) - 1}{z^2} \quad \parallel \cos(z) = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots \quad (\text{Taylorreihe})$$

$$\lim_{z \rightarrow 0} \frac{-\frac{z^2}{2!} + \frac{z^4}{4!} - \dots}{z^2} = \underline{\underline{-\frac{1}{2}}}$$

$$iii) \quad \lim_{z \rightarrow 0} \frac{\sin(z)}{\bar{z}} \quad \parallel z = x + iy$$

$$\lim_{z \rightarrow 0} \frac{\sin(x + iy)}{x - iy} \quad \parallel \text{Wir müssen den Grenzwert von Re und Im separat betrachten}$$

$$\lim_{x \rightarrow 0} \frac{\sin(x + iy)}{x - iy} \quad \parallel y=0$$

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1 \quad \parallel \lim_{x \rightarrow 0} \frac{\sin(x)'}{x'} = \frac{\cos(x)}{1} = 1 \quad (\text{Bernoulli - l'Hôpital})$$

$$\lim_{y \rightarrow 0} \frac{\sin(x + iy)}{x - iy} \quad \parallel x=0$$

$$\lim_{y \rightarrow 0} \frac{\sin(iy)}{-iy} = \lim_{y \rightarrow 0} \frac{i \cdot \sin(y)}{-iy} = \lim_{y \rightarrow 0} -\frac{\sin(y)}{y} = -1 \quad \parallel \text{Bernoulli - l'Hôpital}$$

$$\Rightarrow z_0 = 1 - i \neq 0 \quad \parallel \text{Somit existiert der Limes nicht.}$$

Aufgabe 1

a) i) $\underline{e^i = \cos(1) + i \sin(1)}$

ii) $e^{1-2i} = e \cdot e^{-2i}$

$e^{-2i} = \cos(-2) + i \sin(-2)$

$\Rightarrow \underline{e^{1-2i} = e \cos(-2) + e \cdot i \sin(-2)}$

iii) $\underline{\text{Log}(1+i) = \log(\sqrt{2}) + i \frac{\pi}{4}}$

b) i) $\cos(10i) = \cosh(10)$

$\cosh(10) = \frac{e^x + e^{-x}}{2}$

ii) $\sin(5+5i) = \sin(5)\cos(5i) + \cos(5)\sin(5i)$

$\cos(5i) = \cosh(5) = \frac{e^5 + e^{-5}}{2}$

$\sin(5i) = \sinh(5) = \frac{e^5 - e^{-5}}{2}$

$\underline{\sin(5+5i) = \sin(5) \frac{e^5 + e^{-5}}{2} + \cos(5) \frac{e^5 - e^{-5}}{2}}$

iii) $\sin(2-i) = \sin(2)\cos(i) - \cos(2)\sin(i)$

$\cos(i) = \frac{e^1 + e^{-1}}{2}$

$\sin(i) = \frac{e^1 - e^{-1}}{2}$

$\underline{\sin(2-i) = \sin(2) \frac{e^1 + e^{-1}}{2} - \cos(2) \frac{e^1 - e^{-1}}{2}}$

Aufgabe 2

$\text{Log}(z_1 \cdot z_2) \neq \text{Log}(z_1) + \text{Log}(z_2)$

$\Rightarrow \begin{cases} z_1 = -1 \\ z_2 = i \end{cases}$

|| Wir suchen z_1 und z_2 , sodass die Summe der $\text{Arg}()$ $> \pi$ sind aber das $\text{Arg}()$ von $z_1 \cdot z_2$ kleiner als π ist.

Aufgabe 3

$$(\cos(\phi) + i \sin(\phi))^n = \cos(n\phi) + i \sin(n\phi) \quad \parallel \text{Beweis durch Induktion}$$

$$n = 0$$

$$(\cos(\phi) + i \sin(\phi))^0 = \cos(0\phi) + i \sin(0\phi)$$

$$1 = 1 + 0 \quad \checkmark$$

$$n = 1$$

$$(\cos(\phi) + i \sin(\phi))^1 = \cos(\phi) + i \sin(\phi) \quad \checkmark$$

$$n = k + 1$$

$$(\cos(\phi) + i \sin(\phi))^{k+1} = (\cos(\phi) + i \sin(\phi))^k \cdot (\cos(\phi) + i \sin(\phi))$$

$$= (\cos(k\phi) + i \sin(k\phi)) (\cos(\phi) + i \sin(\phi))$$

$$= \cos(k\phi)\cos(\phi) + \cos(k\phi)i\sin(\phi) + i\sin(k\phi)\cos(\phi) - \sin(k\phi)\sin(\phi) \quad \parallel \text{Kosinus \& Sinus Satz}$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin(A+B) = \cos(A)\sin(B) + \cos(B)\sin(A)$$

$$\Rightarrow \cos(k\phi + \phi) + i \sin(k\phi + \phi) = \cos((k+1)\phi) + i \sin((k+1)\phi)$$



DISCLAIMER

Die Notizen zu den Stack Aufgaben sind auf meine Werte angepasst. Die Werte können von deinen Aufgaben abweichen.

Seite 1

Frage 1

$$(8-i)(5i+3) = 40i + 24 + 5 - 3i = 37i + 29$$

$$i(5i+3) = -5 + 3i$$

$$\bar{w} = -5i + 3$$

$$\frac{8-i}{5i+3} = \frac{(8-i)(5i-3)}{-34} = \frac{40i - 24 + 5 + 3i}{-34} = -\frac{43i}{34} + \frac{19}{34}$$

$$|z| \sqrt{8^2 + 1^2} = \sqrt{65}$$

Frage 2

$$\cos(-\frac{\pi}{4}) > 0; \cos(\frac{\pi}{4}) > 0 \Rightarrow \operatorname{Re}(z) > 0$$

$$\sin(-\frac{\pi}{4}) < 0; \sin(\frac{\pi}{4}) > 0 \Rightarrow \operatorname{Im}(z) > 0 \text{ oder } < 0$$

Frage 3

$$(4+2i)(4+2i) = 16 + 8i + 8i - 4 = 12 + 16i$$

$$(12+16i)(4+2i) = 48 + 24i + 64i - 32 = 16 + 88i$$

Bemerkung: Teilweise ist es einfacher potenzierte komplexe Gleichungen
in Polarform zu berechnen.

Serie 2

Die Stackaufgaben für
Serie 2 waren sehr einfach.
Bei Fragen kann man mich
kontaktieren.

Serie 3

Frage 1

$\operatorname{Im}(z) = 0$ || Der Im von z muss 0 sein damit $z \in \mathbb{R}$

$$z = e^{\frac{3\pi}{4}i} \cdot (\sqrt{2} + bi)$$

$$= \left(\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right) \cdot (\sqrt{2} + bi)$$

$$= \sqrt{2} \cos\left(\frac{3\pi}{4}\right) + bi \cos\left(\frac{3\pi}{4}\right) + \sqrt{2} i \sin\left(\frac{3\pi}{4}\right) - b \sin\left(\frac{3\pi}{4}\right) \quad || \operatorname{Im}(z) = 0$$

$$\Rightarrow bi \cos\left(\frac{3\pi}{4}\right) + \sqrt{2} i \sin\left(\frac{3\pi}{4}\right) = 0$$

$$bi \cos\left(\frac{3\pi}{4}\right) = -\sqrt{2} i \sin\left(\frac{3\pi}{4}\right)$$

$$\cancel{bi} \cdot \frac{\sqrt{2}}{2} = \cancel{-\sqrt{2}i}$$

$$b = \frac{2\sqrt{2}}{2}$$

Frage 2

$$c_n = \frac{(4n)!}{(-2-i)^n \cdot (n!)^4}$$

$$\rho = \frac{1}{\limsup_{n \rightarrow \infty} \sqrt[n]{|c_n|}}$$

$$\limsup_{n \rightarrow \infty} \sqrt[n]{\left| \frac{(4n)!}{(-2-i)^n \cdot (n!)^4} \right|} = \limsup_{n \rightarrow \infty} \sqrt[n]{\left| \frac{1}{(-2-i)^n} \cdot \frac{(4n)!}{(n!)^4} \right|}$$

$$(4n)! = (4n)(4n-1)(4n-2) \dots 4 \cdot 3 \cdot 2 \cdot 1$$

Wenn wir die Multiplikation in 4er Gruppen aufteilen:

$$\left. \begin{array}{l} [(4n)(4n-1)(4n-2)(4n-3)] \leq (4n)^4 \\ \vdots \\ [4 \cdot 3 \cdot 2 \cdot 1] \leq (4n)^4 \end{array} \right\} n\text{-Mal}$$

$$\Rightarrow (4n)! \leq (4n)^4 \cdot (4n)^4 \cdot \dots = (4n)^{4n}$$

$$(n!)^4 = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$$

Wenn wir die Multiplikation in 4er Gruppen aufteilen

$$\left. \begin{array}{l} [n \cdot (n-1)(n-2)(n-3)] \leq n^4 \\ \vdots \\ [4 \cdot 3 \cdot 2 \cdot 1] \leq n^4 \end{array} \right\} n\text{-Mal}$$

$$\Rightarrow (n!)^4 \leq n^{4n}$$

Frage 2

$$\lim_{n \rightarrow \infty} \sup \sqrt[n]{\left| \frac{1}{\sqrt{5}} \left| \frac{(4n)!}{(n!)^4} \right| \right|} \leq \sqrt[n]{\left| \frac{1}{\sqrt{5}} \left| \frac{(4n)^{4n}}{(n)^{4n}} \right| \right|} = \sqrt[n]{\left| \frac{1}{\sqrt{5}} \left| \frac{4^{4n} n^{4n}}{n^{4n}} \right| \right|} = \frac{256}{\sqrt{5}}$$

$$R = \frac{1}{\frac{256}{\sqrt{5}}} = \frac{\sqrt{5}}{256}$$

Frage 3

$$z = (-\sqrt{3} + i)^7 \quad || \quad z = r^n \cdot e^{in\phi}$$

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = 2$$

$$\phi = \arctan\left(\frac{1}{-\sqrt{3}}\right) = -\frac{\pi}{6}$$

$$\Rightarrow z = 2^7 \cdot e^{i\frac{7\pi}{6}} = 128 e^{-i\frac{7\pi}{6}}$$

Serie 4

Frage 1

$$z^5 = -\frac{3}{2} + \frac{3^{\frac{3}{2}}}{2} i$$

$$r = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3} = 3$$

$$\phi = \arctan\left(\frac{\frac{3^{\frac{3}{2}}}{2}}{-\frac{3}{2}}\right) + \pi = \frac{2\pi}{3} + 2\pi k$$

$$s_k = \frac{2\pi}{3} + 2\pi k$$

$$\varphi = \frac{2\pi}{15} + \frac{2\pi k}{5} \quad \parallel \text{ mit } 0 \leq k \leq 4$$

$$z_1 = 3^{\frac{1}{5}} \cdot e^{i \cdot \left(\frac{2\pi}{15}\right)}$$

$$z_2 = 3^{\frac{1}{5}} \cdot e^{i \cdot \left(\frac{2\pi}{15} + \frac{2\pi}{5}\right)}$$

$$z_3 = 3^{\frac{1}{5}} \cdot e^{i \cdot \left(\frac{2\pi}{15} + \frac{4\pi}{5}\right)}$$

$$z_4 = 3^{\frac{1}{5}} \cdot e^{i \cdot \left(\frac{2\pi}{15} + \frac{6\pi}{5}\right)}$$

$$z_5 = 3^{\frac{1}{5}} \cdot e^{i \cdot \left(\frac{2\pi}{15} + \frac{8\pi}{5}\right)}$$

Frage 2

$$\text{Log}\left(2^{\frac{3}{2}} + 2^{\frac{3}{2}} i\right) \parallel \text{Log}(z) = \log(|z|) + \text{Arg}(z) \cdot i$$

$$|z| = \sqrt{\left(2^{\frac{3}{2}}\right)^2 + \left(2^{\frac{3}{2}}\right)^2} = \sqrt{2^3 + 2^3} = \sqrt{16} = 4$$

$$\text{Arg}(z) = \frac{\pi}{4}$$

$$\Rightarrow \log(4) + \frac{\pi}{4} i$$